Volume of Cone Problem **Name _____**

In this activity we will explore the **volumes of cones** of different shapes. You will need to know the following geometry formulas to do this activity

Circumference of a circle = _____

Volume of a cone = _____

Part 1: The Physical Model

The circle you were given has a radius of 10 cm. Its **circumference** is _____ cm. The length of string your group is using has a length of _____ cm. (All groups have different lengths of string.)

Directions:

1. Cut out the circle.
2. Place the string on the circumference of the circle and mark the circle at the ends of the piece of string.
3. Remove the piece of string and draw straight line segments connecting these marks with the center of the circle forming a sector .
4. Cut out this sector of the circle and put it away. (Don't throw it away!)
5. Form a cone by taping the remaining part of the circle with the cut edges touching, but not overlapping.
6. Draw your cone below.

7. The radius of the original circle was 10 cm. What part of the cone has a length of 10 cm? Label this part of your drawing above 10 cm.

In order to find the **volume of the cone** we need to find the _____ and the _____. Let's see if we can do this. Start by finding the **circumference of the circle** which is the **base of the cone**. (Don't measure it! Think about how it was formed.) The **circumference** is _____ cm. (Round all decimals to 2 places). Use this value to find the **radius of the base** (which is the radius of the cone). The **radius** is _____ cm. Put this value where it belongs in your drawing above. Now we need to find the **height of the cone**. Explain how we can do this.

_____

Label the height of your cone **H.** Write and solve the equation you would use to find **H**.

The **height** of the cone is _____ cm. Put this value where it belongs in your drawing above.

We now have enough information to find the **volume of the cone**. Find it! Put all of your values into the volume formula above and calculate it.

Volume = _____ = _____.

Summarize the information about your cone below.

Arc length (of cut out sector)	Radius of cone	Height of cone	Volume of cone
_____	_____	_____	_____

Share this information with the rest of the class by filling in the chart on the board.

Part 2: The Graphical Model

We want to explore the relationship between the **Arc Length** and the **Volume of the Cone**. Put the results of some of your classmates in the chart below. (The arc length values do not have to be in order.)

Arc Length											
Volume of Cone											

Put the **Arc Length** values into **L1** of your calculator and **Volume** into **L2**. Make a scatterplot of this relationship and sketch the graph below.

xmin: _____
xmax: _____
ymin: _____
ymax: _____

Part 3: Algebraic Model

We want to find the function which will go through the data points of the scatterplot. (This is the exciting part and when you are finished you will see the most complicated function you have ever seen!) Redraw your cone below and label the **slant height** 10 cm again.

Our **function** is going to define the **volume** as a function of **x**, where **x** is the **arc length** of the cut out sector. To do this we are going to follow the same steps that we did for your specific cone, only begin by letting the length of the string be **x**. Start by finding the **circumference of the circle** which is the **base of the cone**. (Think about how you found it above.) The **circumference** (in terms of **x**) is _____.
Use this expression to find an expression for the **radius of the base** (which is the radius of the cone).
The **radius** is _____. Put this expression where it belongs in your drawing above.
Now we need to find the **height of the cone**. Again, if the height of the cone is **H,** write an equation you would use to find **H** below, and solve this equation for **H**. (Do not try to simplify the expression!)

The **height** of the cone in terms of **x** is _____. Put this expression where it belongs in your drawing above.
We now have enough information to find the function for the **volume of the cone**. Put your values into the volume formula and write a function **V(x)** which represents the **volume of the cone**.

V(x) = _____

(Was I correct? Is this the "messiest" function you've ever seen?) Well it's time to see if your function is correct. Carefully, put your volume function into **Y1** of your calculator, cross your fingers, and graph it. If your function is correct, answer the questions below.

Part 4: Questions

1. What values of **x** make sense to the problem situation?
_____

2. What arc length will give us the maximum volume? _____

3. What is the maximum volume? _____

Note: You are going to be working with this function later today in your homework assignment. And we don't want to enter it again! So, let's save the equation, the window settings (and the mode) in a Graph Data Base. Then we can recall it when we need it.

Part 5: An Extension of the Cone Problem

(Note: Turn off your scatterplot, if you have not already done so.)

In the problem you solved above, you cut a sector out of a circle with a radius of 10 cm. Let's now use the cut out sector that we put away, and bring together its edges and make a second cone out of it. The problem to be solved now is to maximize the **sum of the volumes of the two cones**. To do this we need to find the volume of the second cone.

In terms of **x**, the arc length of the cut out sector, this cone has a **base** with **circumference** _____ cm, and, therefore, the **radius** of this cone is _____ cm. Find the **height** of the cone (the same way you did for the other cone). The **height** is _____ cm. Now that we know the **radius** and the **height** of the cone, we can write an expression for the volume of the second cone.

$V_2(x)$ = _____

Add this expression to your **original volume function** and then graph it. Sketch your graph below.

Xmin: _____
Xmax: _____
Ymin: _____
Ymax: _____

This function actually has **two** maximum points. To see them you may need to adjust your window. Find the size of the arc length **x** that will cause the **sum of the volumes** of the two cones to be a **maximum**. These **arc lengths** would be _____ cm and _____ cm. The **maximum volume** would be _____.